QUESTÃO 1 – TE

PGFIS – 2014/01

Eletromagnetismo

Q1 2,5 a) $\vec{A} = \alpha \vec{r} \times \vec{B}_0$

$\vec{B}_0 =$ constante

$$\vec{\nabla} \times \vec{A} = \alpha \vec{\nabla} \times (\vec{r} \times \vec{B}_0) = \alpha \Big[ \underbrace{(\vec{B}_0 \cdot \vec{\nabla}) \vec{r}}_{-3B_0\hat{z}} - \underbrace{(\vec{r} \cdot \vec{\nabla}) \vec{B}_0}_{0} + \underbrace{\vec{r} (\vec{\nabla} \cdot \vec{B}_0)}_{0} - \vec{B}_0 \underbrace{(\vec{\nabla} \cdot \vec{r})}_{=3} \Big]$$

$$B_0 \frac{\partial}{\partial z} \vec{r} = B_0 \hat{z}$$

$$\vec{\nabla} \times \vec{A} = -2\alpha B_0 \hat{z} \Rightarrow \boxed{\alpha = -\frac{1}{2}}$$

$$\Downarrow$$

$$\boxed{\vec{\nabla} \times \vec{A} = \vec{B}_0}$$

$$\vec{\nabla} \cdot \vec{A} = \Big[ \underbrace{\vec{B}_0 \cdot (\vec{\nabla} \times \vec{r})}_{=0} - \underbrace{\vec{r} \cdot (\vec{\nabla} \times \vec{B}_0)}_{=0} \Big] \alpha$$

$\vec{\nabla} \cdot \vec{A} = 0$
(Gauge de Coulomb)

2,5 b) $\vec{A}' = \vec{A} + \vec{C}(r)$

$$\vec{\nabla} \times \vec{A}' = \vec{\nabla} \times \vec{A} \Rightarrow \vec{\nabla} \times \vec{C} = 0 \Rightarrow \vec{C} = \vec{\nabla}\phi$$

$$(\vec{\nabla} \cdot \vec{A}' \neq \vec{\nabla} \cdot \vec{A})$$

Exemplo: $\vec{C} = -\frac{\vec{r}}{r^3} = \vec{\nabla}\left(\frac{1}{r}\right)$

QUESTÃO 2 – TE

**Solution:** In the diagram, ($d\mathbf{l}' \times \hat{\mathbf{\mathscr{r}}}$) points *out* of the page, and has the magnitude

$$dl' \sin\alpha = dl' \cos\theta.$$

Biot – Savart law

$$\boxed{\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l}' \times \hat{\mathscr{r}}}{\mathscr{r}^2}}$$ 20%

Also, $l' = s\tan\theta$, so

$$dl' = \frac{s}{\cos^2\theta}\,d\theta,$$ 20%

and $s = \mathscr{r}\cos\theta$, so

$$\frac{1}{\mathscr{r}^2} = \frac{\cos^2\theta}{s^2}.$$ 20%

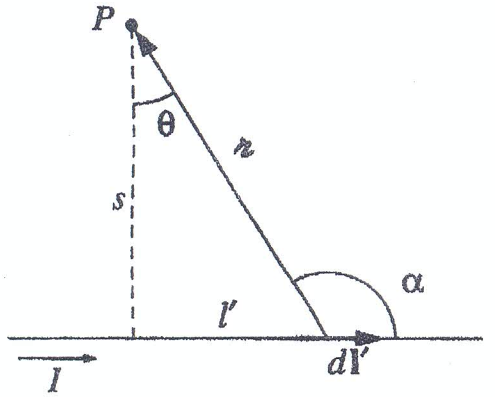


Figure 5.18

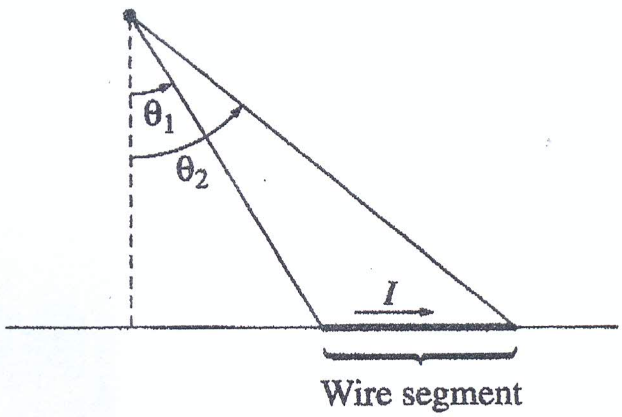


Figure 5.19

Thus

$$\begin{aligned} B &= \frac{\mu_0 I}{4\pi}\int_{\theta_1}^{\theta_2}\left(\frac{\cos^2\theta}{s^2}\right)\left(\frac{s}{\cos^2\theta}\right)\cos\theta\,d\theta \qquad 20\% \\ &= \frac{\mu_0 I}{4\pi s}\int_{\theta_1}^{\theta_2}\cos\theta\,d\theta = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1). \end{aligned} \tag{5.35}$$

Equation 5.35 gives the field of any straight segment of wire, in terms of the initial and final angles $\theta_1$ and $\theta_2$ (Fig. 5.19). Of course, a finite segment by itself could never support a steady current (where would the charge go when it got to the end?), but it might be a *piece* of some closed circuit, and Eq. 5.35 would then represent its contribution to the total field. In the case of an infinite wire, $\theta_1 = -\pi/2$ and $\theta_2 = \pi/2$, so we obtain

$$B = \frac{\mu_0 I}{2\pi s}. \qquad 20\% \tag{5.36}$$

20% → *The symbol 20% represents the percentage assigned to highlighted this issue, ie, the amount that will be credited to the candidate who has agreed to said passage.*