

QUESTÃO 1 - TE

PGFIS - 2014/01

Eletromagnetismo

Q1 2k a)

$$\vec{A} = \alpha \vec{r} \times \vec{B}_0$$

$\vec{B}_0 = \text{constante}$

$-3B_0 \hat{z}$

$$\vec{\nabla} \times \vec{A} = \alpha \vec{\nabla} \times (\vec{r} \times \vec{B}_0) = \alpha \left[\underbrace{(\vec{B}_0 \cdot \vec{\nabla}) \vec{r}}_0 - \underbrace{(\vec{r} \cdot \vec{\nabla}) \vec{B}_0}_0 + \underbrace{\vec{r} (\vec{\nabla} \cdot \vec{B}_0)}_{-3B_0 \hat{z}} - \underbrace{\vec{B}_0 (\vec{\nabla} \cdot \vec{r})}_{=3} \right]$$

$$B_0 \frac{\partial \vec{r}}{\partial z} = B_0 \hat{z}$$

$$\vec{\nabla} \times \vec{A} = -2\alpha B_0 \hat{z} \Rightarrow \boxed{\alpha = -\frac{1}{2}}$$

$$\boxed{\vec{\nabla} \times \vec{A} = \vec{B}_0}$$

$$\vec{\nabla} \cdot \vec{A} = \left[\underbrace{\vec{B}_0 \cdot (\vec{\nabla} \times \vec{r})}_{=0} - \underbrace{\vec{r} \cdot (\vec{\nabla} \times \vec{B}_0)}_{=0} \right] \alpha$$

$\vec{\nabla} \cdot \vec{A} = 0$
(Gauge de Coulomb)

2k b) $\vec{A}' = \vec{A} + \vec{C}(\vec{r})$

$$\vec{\nabla} \times \vec{A}' = \vec{\nabla} \times \vec{A} \Rightarrow \vec{\nabla} \times \vec{C} = 0 \Rightarrow \vec{C} = \vec{\nabla} \phi$$

$(\vec{\nabla} \cdot \vec{A}') = \vec{\nabla} \cdot \vec{A}$ Exemplo $\vec{C} = -\frac{\vec{r}}{r^3} = \vec{\nabla} \left(\frac{1}{r} \right)$

QUESTÃO 2 - TE

Solution: In the diagram, $(d\vec{l}' \times \hat{n})$ points out of the page, and has the magnitude

$$dl' \sin \alpha = dl' \cos \theta.$$

Biot - Savart law

Also, $l' = s \tan \theta$, so

$$dl' = \frac{s}{\cos^2 \theta} d\theta, \quad (20\%)$$

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}' \times \hat{n}}{r^2}$$

and $s = r \cos \theta$, so

$$\frac{1}{r^2} = \frac{\cos^2 \theta}{s^2}. \quad (20\%)$$

(20%)

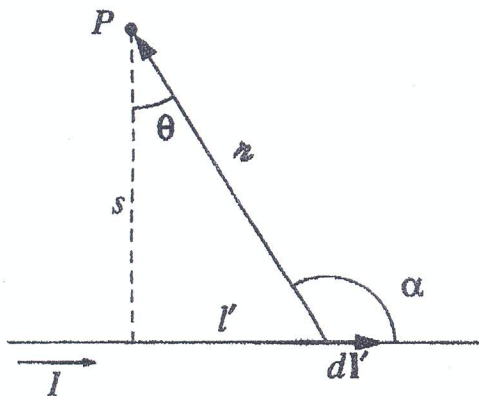


Figure 5.18

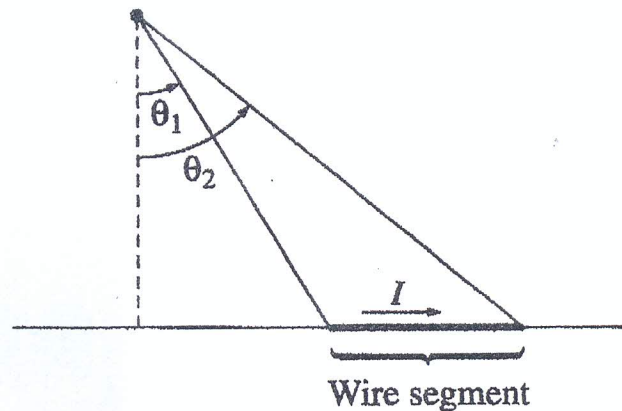


Figure 5.19

Thus

$$B = \frac{\mu_0 I}{4\pi} \int_{\theta_1}^{\theta_2} \left(\frac{\cos^2 \theta}{s^2} \right) \left(\frac{s}{\cos^2 \theta} \right) \cos \theta d\theta \quad (20\%)$$

$$= \frac{\mu_0 I}{4\pi s} \int_{\theta_1}^{\theta_2} \cos \theta d\theta = \frac{\mu_0 I}{4\pi s} (\sin \theta_2 - \sin \theta_1). \quad (5.35)$$

Equation 5.35 gives the field of any straight segment of wire, in terms of the initial and final angles θ_1 and θ_2 (Fig. 5.19). Of course, a finite segment by itself could never support a steady current (where would the charge go when it got to the end?), but it might be a piece of some closed circuit, and Eq. 5.35 would then represent its contribution to the total field. In the case of an infinite wire, $\theta_1 = -\pi/2$ and $\theta_2 = \pi/2$, so we obtain

$$B = \frac{\mu_0 I}{2\pi s}. \quad (20\%) \quad (5.36)$$

(20%) \rightarrow The symbol (20%) represents the percentage assigned to highlighted this issue, ie, the amount that will be credited to the candidate who has agreed to said passage.

